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Short Communication

Nonlinear dynamics of atomic force microscope with PI feedback

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Abstract

Frequency modulation of an atomic force microscope is based on a PI control law, which keeps the amplitude equal to a desired value. This paper analyses the stability and performance of the closed-loop nonlinear dynamics of AFM. Numerical results are presented to corroborate the validity of theoretical analysis.

1. Introduction

Atomic force microscope [1] has provided the foundation for the development of nanotechnology. The principle of an atomic force microscope (AFM) is based on the changes in vibration characteristics of a cantilever beam due to forces between cantilever tip and the sample. Because these forces are inter-atomic in nature, it is not required that the sample be an electrically conducting surface. As a result, AFM is applicable to conducting and nonconducting surfaces as well.

Binning and Quate [1] used changes in the amplitude of vibration due to changes in the force between tip and sample at a constant frequency. But, for a higher sensitivity, Albrecht et al. [2] proposed to measure changes in the frequency of vibration at a constant amplitude to determine the force between the cantilever tip and sample. To maintain the constant amplitude of vibration,

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cantilever vibration is feedback. A schematic diagram of the feedback system is shown by Holscher [3].

There are a number of papers dealing with the dynamic response of the AFM cantilever. The system dynamics is nonlinear because of the nature of force between tip and sample. In general, the method of harmonic balance has been applied to compute the frequency shift as a function of the interaction force between tip and sample.

In this paper, the method of slowly varying parameters [4] has been used to examine the nonlinear dynamics of AFM under PI feedback law. The method of slowly varying parameters yields the dynamics of amplitude and phase of vibration, and therefore has been used to analyse the stability and performance of the PI control law. In steady state, the method of slowly varying parameters leads to same result as that obtained by the method of harmonic balance.

2. Analysis

Consider the model shown in Fig. 1. The differential equation of motion is

$$-k_c(y-x)-\beta_c(\dot{y}-\dot{x})-F_{\text{tsa}}=m_c\ddot{y}, \tag{1}$$

where F_{tsa} is the attractive force between tip and sample given [5] by

$$F_{\text{tsa}}(y,z)=\begin{cases}\dfrac{HR}{6(y+z)^2}, & y+z\geqslant a_0,\\ \dfrac{HR}{6a_0^2}-\dfrac{4}{3}E^*\sqrt{R}(a_0-(y+z))^{3/2}, & y+z<a_0,\end{cases} \tag{2}$$

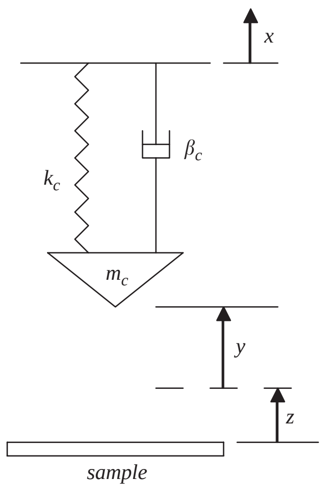

Fig. 1. Single-degree-of-freedom model.

where H, R and a_0 are Hamaker constant, tip radius and intermolecular distance, respectively. The effective elastic modulus E^* is defined as

$$E^* = \left[\frac{1-v_t^2}{E_t} + \frac{1-v_s^2}{E_s}\right]^{-1}, \tag{3}$$

where E_t and E_s are Young's moduli of elasticity for tip and sample, respectively. And, v_t and v_s are Poisson's ratios of tip and sample, respectively. When the separation between the tip and the sample is greater than a certain distance a_0, the force is called van der Waals force, and is attractive in nature. For the tip–sample distance less than a_0, the attractive force is reduced due to Pauli and ionic repulsion [5].

Rearranging Eq. (1),

$$\ddot{y} + 2\xi_c\omega_c\dot{y} + \omega_c^2 y + \omega_c^2\frac{F_{\text{tsa}}}{k_c} = F_{\text{ext}}(t), \tag{4}$$

where

$$F_{\text{ext}}(t) = \omega_c^2 x(t) + 2\xi_c\omega_c\dot{x}, \tag{5}$$

$$\omega_c^2 = \frac{k_c}{m_c}, \tag{6}$$

$$\xi_c = \frac{\beta_c}{2m_c\omega_c}. \tag{7}$$

Assume that the solution of Eq. (4) is represented as

$$y(t) = Y(t)\sin(\omega t + \psi(t)), \tag{8}$$

where the amplitude $Y(t)$ and the phase $\psi(t)$ are slowly varying functions of time. The frequency ω of the response is not known a priori.

Differentiating Eq. (8) with respect to time,

$$\dot{y} = \omega Y\cos\beta + \dot{Y}\sin\beta + \dot{\psi}Y\cos\beta. \tag{9}$$

Now, it is assumed [4] that

$$\dot{Y}\sin\beta + \dot{\psi}Y\cos\beta = 0. \tag{10}$$

Then, from Eqs. (9) and (10),

$$\dot{y} = \omega Y\cos\beta. \tag{11}$$

Differentiating Eq. (11) again,

$$\ddot{y} = \omega\dot{Y}\cos\beta - \omega^2 Y\sin\beta - \omega Y\dot{\psi}\sin\beta. \tag{12}$$

Substituting Eqs. (11) and (12) into Eq. (4),

$$(\omega_c^2 - \omega^2)Y\sin\beta + \omega\dot{Y}\cos\beta - \omega Y\dot{\psi}\sin\beta + 2\xi_c\omega_c\omega Y\cos\beta + \omega_c^2\frac{F_{\text{tsa}}}{k_c} = F_{\text{ext}}(t). \tag{13}$$

Solving for $\dot{Y}$ and $\dot{\psi}$ from Eqs. (10) and (13),

$$\omega\dot{Y} = -\tfrac{1}{2}(\omega_c^2 - \omega^2)Y\sin 2\beta - \xi_c\omega_c\omega Y(1+\cos 2\beta) - \omega_c^2\frac{F_{\rm tsa}}{k_c}\cos\beta + F_{\rm ext}(t)\cos\beta, \tag{14}$$

$$\omega Y\dot{\psi} = \tfrac{1}{2}(\omega_c^2 - \omega^2)Y(1-\cos 2\beta) + \xi_c\omega_c\omega Y\sin 2\beta + \omega_c^2\frac{F_{\rm tsa}}{k_c}\sin\beta - F_{\rm ext}(t)\sin\beta. \tag{15}$$

Since the amplitude $Y(t)$ and the phase $\psi(t)$ are slowly varying functions of time, it is assumed that they are constants during a cycle of oscillation; i.e., β varying from 0 to 2π. Integrating Eqs. (14) and (15) from 0 to 2π with this assumption,

$$2\pi\omega\dot{Y} = -2\pi\xi_c\omega_c\omega Y - \omega_c^2\int_0^{2\pi}\frac{F_{\rm tsa}}{k_c}\cos\beta\,{\rm d}\beta + \int_0^{2\pi}F_{\rm ext}(t)\cos\beta\,{\rm d}\beta, \tag{16}$$

$$2\pi\omega Y\dot{\psi} = \pi(\omega_c^2 - \omega^2)Y + \omega_c^2\int_0^{2\pi}\frac{F_{\rm tsa}}{k_c}\sin\beta\,{\rm d}\beta - \int_0^{2\pi}F_{\rm ext}(t)\sin\beta\,{\rm d}\beta. \tag{17}$$

Control law:

According to Gotsmann et al. [6], the PI control law is defined as

$$F_{\rm ext}(t) = Qy(t-\tau), \tag{18}$$

where

$$Q = q_p(Y(t) - Y_0) + q_I\int_0^t (Y(t') - Y_0)\,{\rm d}t'. \tag{19}$$

Substituting Eq. (18) into Eqs. (16) and (17),

$$2\pi\omega\dot{Y} = -2\pi\xi_c\omega_c\omega Y - \omega_c^2\int_0^{2\pi}\frac{F_{\rm tsa}}{k_c}\cos\beta\,{\rm d}\beta - \pi QY\sin\omega\tau, \tag{20}$$

$$2\pi\omega Y\dot{\psi} = \pi(\omega_c^2 - \omega^2)Y + \omega_c^2\int_0^{2\pi}\frac{F_{\rm tsa}}{k_c}\sin\beta\,{\rm d}\beta - \pi QY\cos\omega\tau. \tag{21}$$

Select

$$\tau = \frac{\pi}{2\omega}. \tag{22}$$

Substituting Eq. (22) into Eqs. (20) and (21),

$$2\pi\omega\dot{Y} = -2\pi\xi_c\omega_c\omega Y - \omega_c^2\int_0^{2\pi}\frac{F_{\rm tsa}}{k_c}\cos\beta\,{\rm d}\beta - \pi QY, \tag{23}$$

$$2\pi\omega Y\dot{\psi} = \pi(\omega_c^2 - \omega^2)Y + \omega_c^2\int_0^{2\pi}\frac{F_{\rm tsa}}{k_c}\sin\beta\,{\rm d}\beta. \tag{24}$$

Define

$$W(t) = \int_0^t (Y(t') - Y_0)\,{\rm d}t'. \tag{25}$$

Then,

$$\dot{W} = Y(t) - Y_0 \tag{26}$$

and

$$Q = q_p(Y(t) - Y_0) + q_I W(t). \tag{27}$$

Substituting Eq. (27) into Eq. (23), the averaged nonlinear closed-loop dynamics can be represented in the state space-form as follows:

$$2\pi\omega \dot{Y} = -2\pi\xi_c\omega_c\omega Y - \omega_c^2 \int_0^{2\pi} \frac{F_{\text{tsa}}}{k_c} \cos\beta \,\mathrm{d}\beta - \pi q_p(Y(t) - Y_0)Y - \pi q_I W(t) Y, \tag{28}$$

$$\dot{W} = Y(t) - Y_0, \tag{29}$$

$$2\pi\omega Y\dot{\psi} = \pi(\omega_c^2 - \omega^2)Y + \omega_c^2 \int_0^{2\pi} \frac{F_{\text{tsa}}}{k_c} \sin\beta \,\mathrm{d}\beta. \tag{30}$$

Note that Eqs. (28)–(30) are in the form of a state-space model [7].

In steady state,

$$\dot{Y} = 0, \quad \dot{W} = 0, \quad \dot{\psi} = 0. \tag{31}$$

Hence, from Eq. (29), in steady state,

$$Y(t) = Y_0 \tag{32}$$

and from Eq. (30),

$$\omega_0 = \omega_c \left[1 + \frac{1}{\pi Y_0} \int_0^{2\pi} \frac{F_{\text{tsa}}}{k_c} \sin\beta \,\mathrm{d}\beta \right]^{0.5} \tag{33}$$

and from Eq. (28),

$$W_0 = \frac{1}{\pi q_I Y_0} \left[-2\pi\xi_c\omega_c\omega Y_0 - \omega_c^2 \int_0^{2\pi} \frac{F_{\text{tsa}}}{k_c} \cos\beta \,\mathrm{d}\beta \right]. \tag{34}$$

Results from numerical integration indicate that

$$\int_0^{2\pi} \frac{F_{\text{tsa}}}{k_c} \cos\beta \,\mathrm{d}\beta \approx 0. \tag{35}$$

In Eqs. (33) and (34), integrals are defined for $Y(t) = Y_0$. To examine the stability of the steady-state solution, small perturbations $\hat{Y}(t)$ and $\hat{W}(t)$ are introduced; i.e.,

$$Y(t) = Y_0 + \hat{Y}(t) \tag{36}$$

and

$$W(t) = W_0 + \hat{W}(t). \tag{37}$$

Substituting Eqs. (36) and (37) into Eqs. (28) and (29) and using Eq. (35), the following equations are obtained after linearization:

$$\begin{bmatrix} \dot{\hat{Y}} \\ \dot{\hat{W}} \end{bmatrix} = \begin{bmatrix} -\frac{q_p Y_0}{2\omega_0} & -\frac{q_I Y_0}{2\omega_0} \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \hat{Y} \\ \hat{W} \end{bmatrix}. \quad (38)$$

The characteristic equation for the linearized dynamic equations (38) is

$$\lambda^2 + \frac{q_p Y_0}{2\omega_0}\lambda + \frac{q_I Y_0}{2\omega_0} = 0. \quad (39)$$

Hence, the necessary and sufficient conditions for the stability are

$$q_p > 0 \quad \text{and} \quad q_I > 0. \quad (40)$$

Natural frequency and damping ratio for the amplitude dynamics can be defined as

$$\omega_Y = \sqrt{\frac{q_I Y_0}{2\omega_0}} \quad \text{and} \quad \xi_Y = \frac{q_p}{2\sqrt{q_I}}\sqrt{\frac{Y_0}{2\omega_0}}. \quad (41)$$

For a critically damped closed-loop system,

$$q_p = 2\sqrt{2}\sqrt{\frac{q_I \omega_0}{Y_0}}. \quad (42)$$

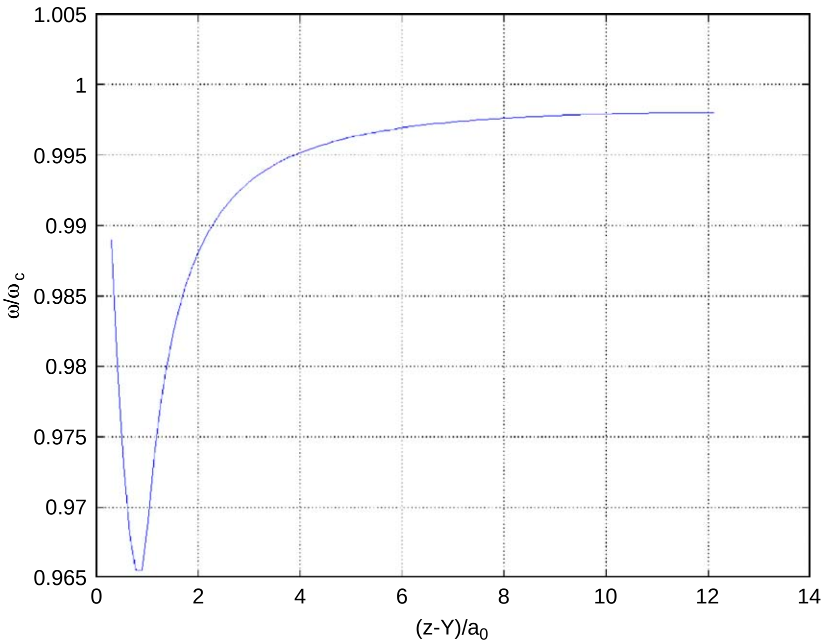

Fig. 2. Frequency versus minimum airgap.

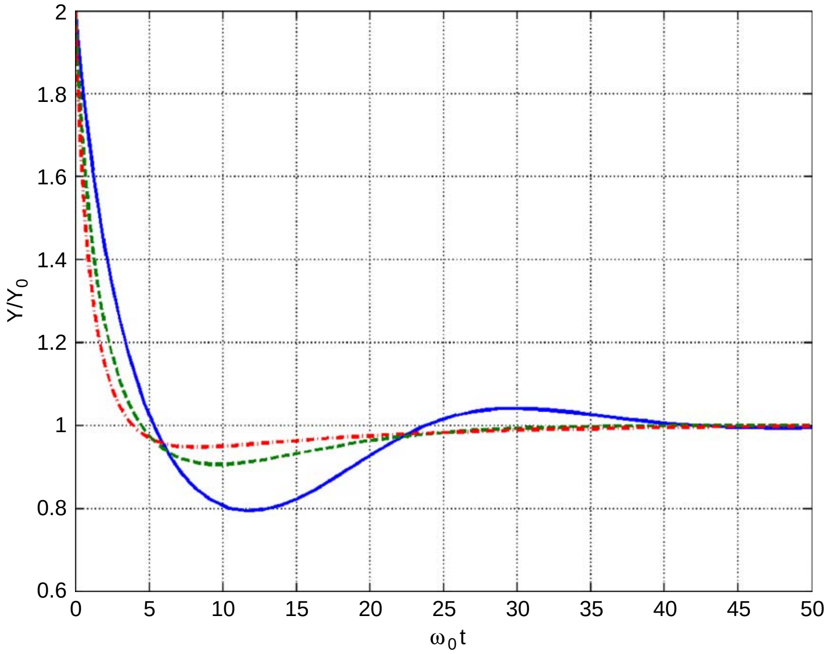

Fig. 3. Amplitude versus time under PI feedback control: ——, $\xi_Y = 0.5$; - - -, $\xi_Y = 1$; - · -, $\xi_Y = 1.5$.

3. Numerical results

Numerical results are obtained for following parameter values: $m_c = 2.0678 \times 10^{-12}$ kg, $k_c = 10$ N/m, $\xi_c = 0.001$, $z = 2 \times 10^{-9}$ m, $a_0 = 0.166 \times 10^{-9}$ m, $R = 15 \times 10^{-9}$ m, $E_t = 1.29 \times 10^{11}$ N/m^2, $v_t = 0.28$, $E_s = 7 \times 10^{10}$ N/m^2, $v_s = 0.3$.

In Fig. 2, the frequency ω is plotted as a function of the minimum airgap during the tip oscillation, which equals $z - Y$. As Y increases, the minimum airgap decreases, and the frequency decreases because of a decrease in the equivalent spring stiffness. After the minimum gap decreases beyond a_0, frequency increases due to a reduction in attractive force.

In Fig. 3, responses from the numerical integration of Eqs. (28) and (29) are plotted. It is seen that the perturbation in the amplitude from $Y_0 = 1.5 \times 10^{-9}$ m dies out, which corroborates the stability of the closed-loop system. Furthermore, as predicted by the linearized analysis, the response is underdamped ($\xi_Y = 0.5$), critically damped ($\xi_Y = 1.0$) or overdamped ($\xi_Y = 1.5$).

4. Conclusions

The method of slowly varying parameters has been successfully applied to derive the state-space model of nonlinear dynamics of an AFM cantilever under PI feedback control, where feedback coefficients are functions of the amplitude of vibration. With the help of this state-space model, the stability and performance of the control system have been analysed.

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